

ON THE MEASUREMENT OF THE CRACK TIP STRESS FIELD AS A MEANS OF DETERMINING ΔK_{eff} UNDER CONDITIONS OF FATIGUE CRACK CLOSURE

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SUMMARY

The optical method of caustics has been successfully extended to enable stress intensity factors as low as $1\text{MPa}\sqrt{\text{m}}$ to be determined accurately for central fatigue cracks in 2024-T3 aluminium alloy test panels. The feasibility of using this technique to study crack closure, and to determine the effective stress intensity factor range, ΔK_{eff} , has been investigated. Comparisons have been made between the measured values of stress intensity factor, K_{caus} , and corresponding theoretical values, K_{theo} , for a range of fatigue cracks grown under different loading conditions. The values of K_{caus} and K_{theo} were in good agreement at maximum stress, where the cracks are fully open, while K_{caus} exceeded K_{theo} at minimum stress, due to crack closure. However, the levels of crack closure and values of ΔK_{eff} obtained could not account for the variations of crack growth rate with loading conditions. It is concluded that the values of ΔK_{eff} , based on caustic measurements in a $1/\sqrt{r}$ stress field well outside the plastic zone, do not fully reflect local conditions which control crack tip behaviour.

INTRODUCTION

Fatigue crack closure under cyclic tensile loading was discovered by Elber in 1970 [1,2]. Since that time crack closure has been extensively studied and widely used to explain fatigue crack growth behaviour [3]. The most successful fatigue crack growth prediction models are based on crack closure. In these models the rate of crack growth is controlled by the effective stress intensity factor range, ΔK_{eff} , which is defined as:

$$\Delta K_{eff} = K_{max} - K_{cl} \quad (1)$$

where K_{cl} is the closure stress intensity factor.

Unfortunately, accurate measurement of crack closure has proved difficult, and different measurement techniques have tended to yield different results [4-7]. Although mechanical compliance measurements have been widely used, with crack closure being determined from deviations from linearity of load versus strain gauge or clip gauge readings, various problems still exist and the accuracy of closure stresses determined in this way remains uncertain.

The optical method of caustics is a relatively new technique where the stress intensity factor is determined by measuring out of plane displacements around the crack tip in a K-field well outside the plastic zone [K-field is defined as a stress field where the dominant term at a distance r from the crack tip is proportional to $K/r^{1/2}$].

This paper describes work which was carried out in order to assess the feasibility of using caustics to determine crack closure and ΔK_{eff} , by comparing measured stress intensity factors, K_{caus} , with calculated theoretical values, K_{theo} . Results obtained for centre cracked aluminium alloy panels, containing fatigue cracks grown under various loading conditions are described and discussed.

CRACK CLOSURE AND THE BACKGROUND TO THE STUDY

Crack closure is understood to occur as a result of several mechanisms. The stretch zone in the wake of the crack caused by monotonic plasticity was the first to be discovered (by Elber [1, 2]). In addition, surface roughness and oxidation of the crack flank are known to produce premature crack closure [8, 9]. Regardless of the closure mechanism, the effect is the same: to reduce the strain amplitude at the crack tip so as to reduce the crack growth rate compared to a closure-free crack.

The effects of closure are considered to reduce ΔK_{eff} (see equation 1), which is often expressed in terms of a factor, U , defined as $\Delta K_{\text{eff}}/\Delta K$. The assumed stress field for an ideal crack (fully open) and a real crack (with closure) are shown schematically in Figure 1. At K_{max} , where both types of crack are open the stress field ahead of the crack is the familiar $1/\sqrt{r}$ singularity K-field. At K_{min} ($R=0$) a closure-free crack would exhibit only a small residual stress field resulting from the compression of the stretched monotonic plastic zone by the surrounding elastic material. In contrast, with a real crack the plastically deformed wake first comes into contact at an applied stress intensity factor above K_{min} . As the applied load is further reduced this wake contact increases until the crack is said to be fully closed. At K_{min} the stress/strain state at the crack tip may be considered to be equivalent to that of an ideal crack at a higher stress intensity factor, referred to as K_{cl} . Thus, it is assumed that the stress field ahead of the crack (outside the plastic zone) is a $1/\sqrt{r}$ K-field of intensity K_{cl} . There is some debate, however, as to whether K_{cl} corresponds to the point where the crack starts to close or whether damage still occurs below this point. This is borne out by the fact that many researchers disagree as to whether K_{cl} is determined by the point on the compliance curve where non-linearity first occurs or where two linear portions would intersect.

Thus, fatigue of a real crack is assumed to result in a cyclic crack tip strain amplitude and crack growth rate consistent with a closure-free crack with a ΔK equivalent to ΔK_{eff} .

In order to study crack closure and determine ΔK_{eff} , the stress field ahead of the crack tip provides an alternative source of information to compliance measurements. If the stress intensity factor can be experimentally determined from this stress field then comparison with the applied, theoretical, stress

intensity factor through a fatigue cycle should yield a result as shown schematically in Figure 2. As the applied K is reduced from $K_{\max}$ the stress intensity factor experienced by the crack tip falls linearly until the crack starts to close and then decreases non-linearly to K_{cl} at minimum stress.

The optical method of caustics provides a means of experimentally determining the stress intensity factor from the gradient of the stress field ahead of the crack tip.

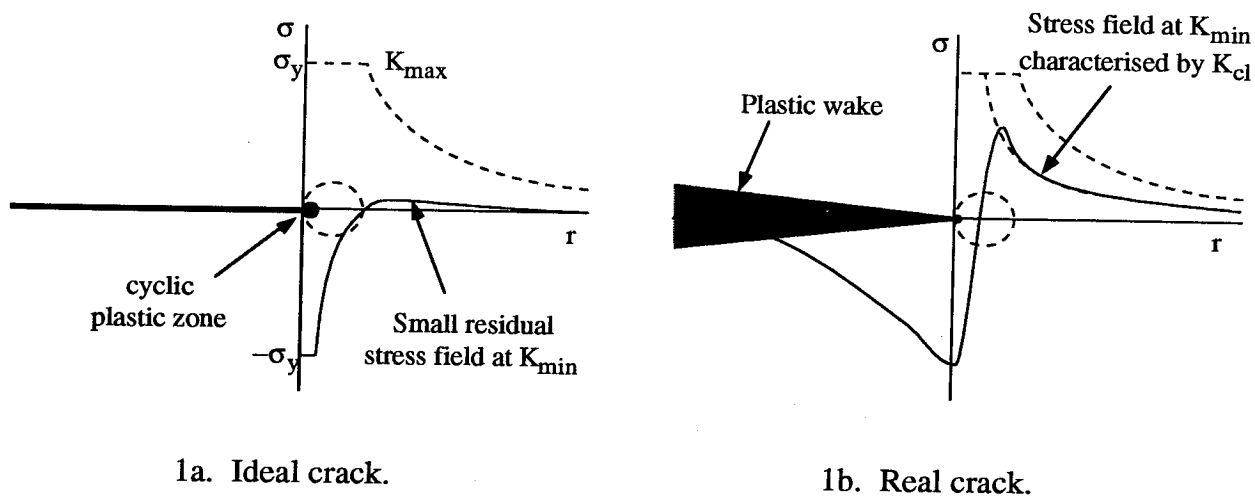


Figure 1. Crack tip stress field at the extremes of the fatigue cycle, with and without crack closure.

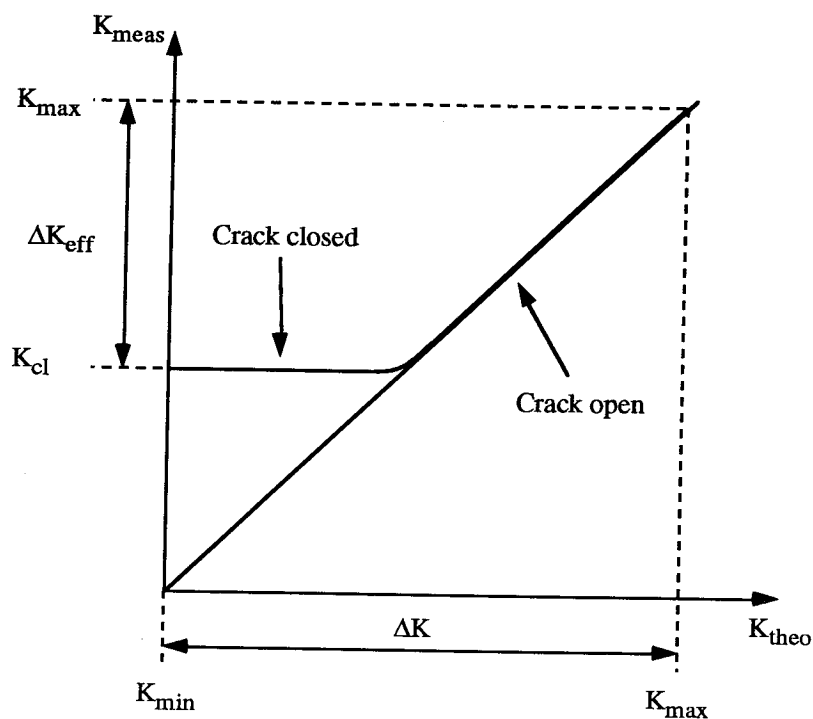


Figure 2. Expected measured K against theoretical K during fatigue.

THE OPTICAL METHOD OF CAUSTICS

A brief explanation of the method of caustics applied to an opaque specimen is as follows. A complete description is found in references [10, 11]. When a plate is subjected to a tensile stress its thickness reduces due to Poisson's effect. If there is a stress gradient the thickness change will be non-uniform, that is, there will be a surface deformation. In the case of a through crack in a parallel sided plate which is subjected to a (mode I) tensile stress, the stress distribution at the crack tip produces a surface deformation as shown schematically in Figure 3. If the surface of the plate is mirrored and illuminated with a spatially coherent beam (such as a laser) then the nature of the surface deformation causes the reflected rays to propagate in a direction as if they had formed a caustic surface behind the specimen as shown in Figure 3.

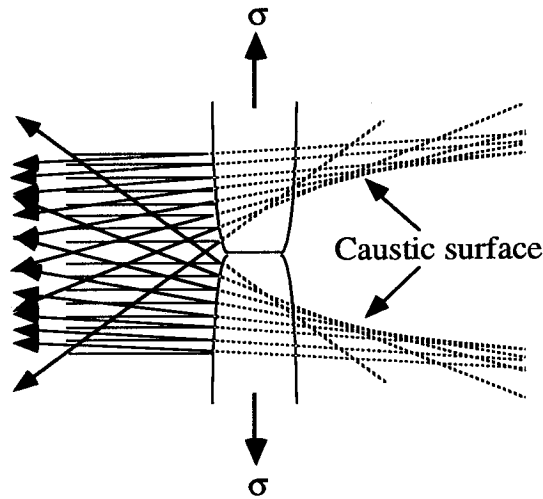


Figure 3. Formation of an optical caustic.

This virtual caustic can be imaged onto a screen in front of the specimen by including a lens in the optical path. The diameter of the caustic, i.e. a cross-section through the virtual caustic surface, for a given specimen is a function of the distance from the specimen surface from which the cross-section is taken (z_0) and the stress intensity factor (K_I) applied to the specimen. The caustic diameter is therefore a measure of the stress intensity factor and, for illumination with a collimated beam, is given by [12]:

$$K_I = \left(\frac{ED^{5/2}}{10.71z_0dv} \right) \quad (2)$$

where E and ν are the material's elastic modulus and Poisson's ratio respectively

D is the transverse diameter of the virtual caustic cross-section

z_0 is the distance between the specimen surface and the plane of caustic

d is the specimen thickness.

The diameter, D' , of the caustic viewed on the screen is related to D by the magnification of the imaging lens and is given by:

$$D = \frac{f}{v-f} D' \quad (3)$$

where f is the focal length of the lens

v is the distance between the lens and the screen

Now, each caustic ring is produced from a unique radius, r_0 , on the specimen, centred around the crack tip, given by [12].

$$r_0 = \left(\frac{3z_0 B \nu K_I}{2\sqrt{2\pi E}} \right)^{\frac{2}{5}} = 0.315D \quad (4)$$

Manipulation of the optical arrangement (lens focal length and screen position) changes the z_0 and therefore changes the r_0 . It is possible, therefore, to choose the measurement radius around the crack tip. Equation (2) is derived assuming the out of plane displacement field is caused by a $1/\sqrt{r}$ plane stress field. By taking a series of measurements at a range of radii from the crack tip the actual stress field's approximation to a K-field can be assessed. It is known that close to the crack tip this assumption breaks down and equation (2) becomes invalid. Within 1.5 times the plane stress plastic zone size plasticity effects impede the measurements [13]. Also, and more importantly, up to a radius of approximately half the specimen thickness the onset of a triaxial stress field affects the measurements [14, 15]. Within the K-dominant zone outside this region the caustic technique can yield accurate stress intensity factor measurements. A $1/\sqrt{r}$ K-field will, therefore, produce caustic measurements which, if presented as a ratio of measured K (K_{caus}) to calculated K (K_{theo}), are shown schematically in Figure 4. Measurements differing from this standard result would indicate a deviation from a K-field.

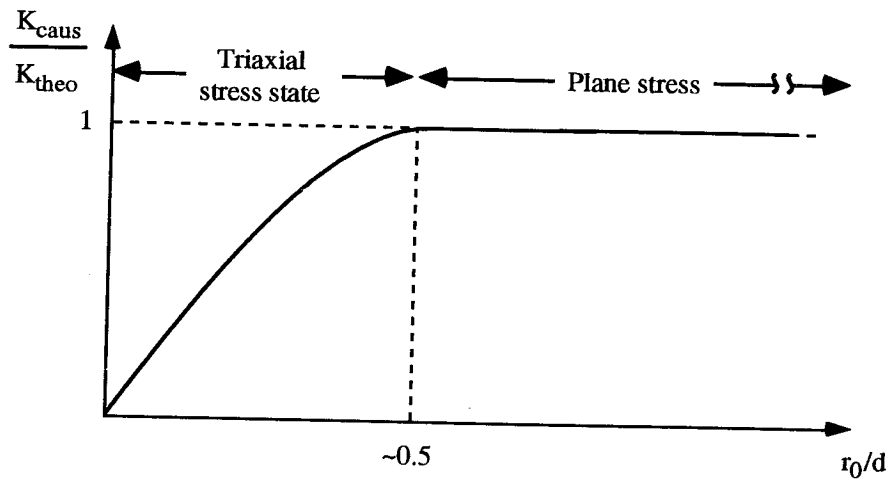


Figure 4. Ideal caustic results for a crack tip K-field.

Despite the fact that the method of caustics has been applied to mode I cracks for almost 30 years surprisingly little research has been conducted on fatigue problems [16-19]. One of the reasons for this is that investigation of fatigue usually requires the measurement of low stress intensity factors where closure is prevalent. The use of caustics at low stress intensity factors is notoriously inaccurate [20] since the Poisson contraction is necessarily very small. However, substantial developments to the technique have been made during this research [20-21] which have facilitated measurements in aluminium alloy at stress intensity factors as low as $1\text{MPa}\sqrt{\text{m}}$. These developments have led to a substantial modification of the technique compared with that used by other researchers. Inherent inaccuracies are overcome by using an interferometrically focused laser to simultaneously illuminate both sides of an optically flat single point diamond machined specimen* together with a CCD camera image processing system† to measure the caustic.

EXPERIMENTS

Experiments were conducted on 6mm thick, 300x160mm, 2024-T3 aluminium alloy centre-cracked panel specimens. This widely used aerospace material has been experimentally tested and theoretically analysed extensively over the past two decades since crack closure was discovered and is known to exhibit significant closure.

Variations in the magnitude of crack closure is used to explain stress ratio effects on crack propagation and also load interaction effects. For this reason the specimens were tested at a range of stress ratios, after a single tensile overload, after a single compressive underload and after a high/low block loading sequence as described below.

Stress intensity factors are determined throughout the fatigue cycle by taking the mean of a series of caustic measurements from within the plateau (plane stress) region of Figure 4.

i) Constant Amplitude Fatigue ($\Delta K=10\text{MPa}\sqrt{\text{m}}$, $R=0.1, 0.3$ and 0.6)

Fatigue cracks grown at the same ΔK , but at a range of stress ratios, were analysed to facilitate comparison between cracks with different levels of crack closure. In addition, a centre-cracked panel was produced by electro-discharge machining a narrow notch (approximately $80\mu\text{m}$ root radius) into the

* The latest developments have recently been submitted for publication in Engineering Fracture Mechanics under the title *High accuracy stress intensity factor measurement using the optical method of caustics* by Wallhead, I.R. and L. Edwards.

† Analysis performed on a Macintosh IIsx computer using the public domain NIH Image program (written by Wayne Rasband at the U.S. National Institutes of Health and available from the Internet by anonymous ftp from [zipper.nimh.nih.gov](ftp://zipper.nimh.nih.gov) or on floppy disc from NTIS, 5285 Port Royal Rd., Springfield, VA 22161, part number PB93-504868).

specimen. It is assumed that such a narrow notch provides a good approximation to a real crack except that there is no plastic wake behind the crack tip and for this reason it is termed an 'ideal crack'. Stress intensity factor measurements were then taken over the unloading half of a fatigue cycle of $\Delta K=10\text{MPa}\sqrt{\text{m}}$, $R=0.1$ so as to compare with a real crack over the same loading cycle.

ii) Single Tensile Overload

A crack grown at a ΔK of $10\text{MPa}\sqrt{\text{m}}$ ($R=0.1$) was given a 60% overload in order to induce an increase in closure and consequent crack growth retardation. Measurements were made pre-overload, at two points within the retardation zone and also after the crack had returned to the pre-overload growth rate.

iii) Single Compressive Underload

A crack grown at a ΔK of $10\text{MPa}\sqrt{\text{m}}$ ($R=0.1$) was given a compressive underload of an equal magnitude to the fatigue cycle maximum tensile load. Again, measurements were made pre-compression and at several points post compression.

iv) High/Low Block Loading

Caustic measurements were taken through a fatigue cycle of $\Delta K=10\text{MPa}\sqrt{\text{m}}$, $R=0.1$ following 5mm of crack growth at $\Delta K=10\text{MPa}\sqrt{\text{m}}$, $R=0.6$. This load sequence exaggerates the closure effect such that the crack is expected to be closed throughout the whole fatigue cycle of the lower block.

RESULTS AND DISCUSSION

i) Constant Amplitude Fatigue ($\Delta K=10\text{MPa}\sqrt{\text{m}}$, $R=0.1$, 0.3 and 0.6)

Figure 5 shows the measured (K_{caus}) against calculated (K_{theo}) stress intensity factors for cracks grown at the range of stress ratios shown and also for an electro-discharge machined ideal crack at $R=0.1$. The dashed 45° line indicates a fully open crack since $K_{\text{caus}}=K_{\text{theo}}$. Deviation from this line therefore shows that crack closure is occurring.

The most striking feature of Figure 5 is the difference between the real and ideal cracks ($R=0.1$). The ideal crack shows good agreement with theory over the whole loading range. The real fatigue crack exhibits a marked departure from theory approaching K_{min} . This effect is considered to be caused by crack closure since the main difference between these specimens is the absence of a plastic stretch zone in the wake of the ideal crack.

Figure 5 also shows that crack closure is more pronounced for cracks grown at $R=0.1$ than at $R=0.3$. No closure was detected for a crack grown at $R=0.6$. ΔK_{eff} was calculated from equation 1, using values

of K_{cl} given by K_{caus} measured at minimum stress. Thus, the values of U ($\Delta K_{eff}/\Delta K$) shown in Table 1 were determined.

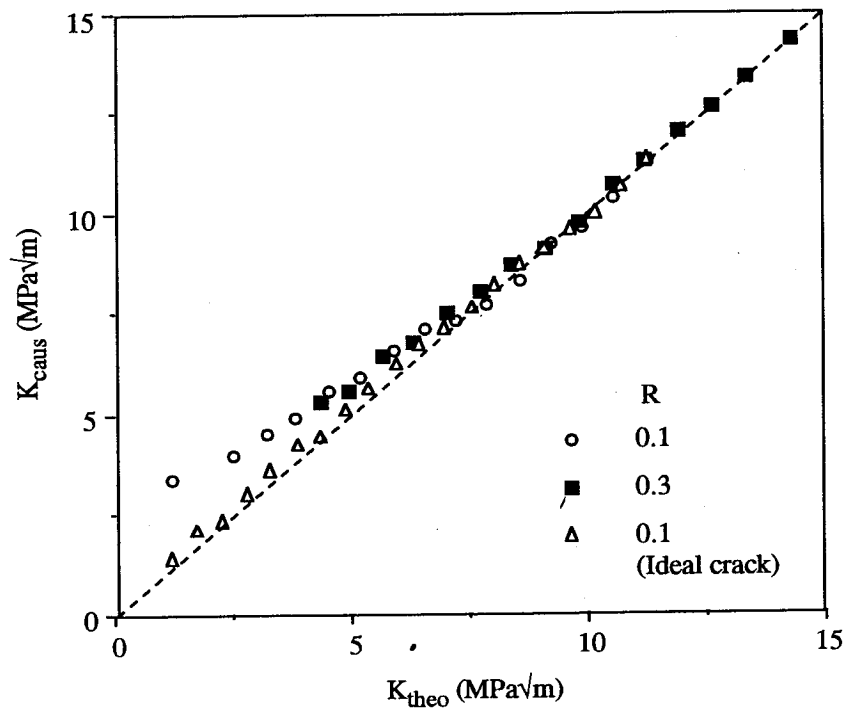


Figure 5 : Constant amplitude fatigue results

Table 1. Calculation of ΔK_{eff} and U

R	K_{max} (MPa√m)	$K_{min}(caus)$ (MPa√m)	ΔK_{eff} (MPa√m)	U (caustic)	U (Newman [23]) $\alpha=2.4$ (see below)
0.1	11.1	3.5	7.6	0.76	0.77
0.3	14.3	5.5	8.8	0.88	0.88

These values can be compared with theoretical predictions from Newman's analytical model [22]. Based on this model Newman has produced crack opening stress equations [23] which are applicable to centre-cracked tension specimens under constant amplitude loading. In addition he presents closure predictions which collapse da/dN v ΔK plots for 2024-T3 aluminium alloy. These equations, which are derived from polynomial fits to a range of results from the model, are as follows:

$$\frac{S_o}{S_{max}} = A_0 + A_1R + A_2R^2 + A_3R^3 \quad \text{for } R \geq 0 \quad (5)$$

and

$$\frac{S_o}{S_{max}} = A_0 + A_1R \quad \text{for } -1 \leq R < 0 \quad (6)$$

The coefficients are:

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2) \left[\cos \left(\frac{\pi S_{\max}}{2\sigma_0} \right) \right]^{1/\alpha} \quad (7)$$

$$A_1 = (0.415 - 0.071\alpha) \left(\frac{S_{\max}}{\sigma_0} \right) \quad (8)$$

$$A_2 = 1 - A_0 - A_1 - A_3 \quad (9)$$

$$A_3 = 2A_0 + A_1 - 1 \quad (10)$$

where S_0 is the crack opening stress

$S_{\max}$ is the maximum stress in the constant amplitude cycle

(Note $S_0/S_{\max}$ could equally be described as $K_0/K_{\max}$)

R is the stress ratio

α is a 'constraint factor' (1 for plane stress, 3 for plane strain)

σ_0 is the material flow stress (average between the yield stress and ultimate tensile strength)

The ratio $S_0/S_{\max}$ can be used to calculate U according to:

$$U = \frac{\Delta K_{\text{eff}}}{\Delta K} = \frac{1 - \frac{S_0}{S_{\max}}}{1 - R} \quad (11)$$

These equations agree with the experimental results provided the (empirically determined) constraint factor, α , has a value of 2.4. In reference [23] Newman identifies a constraint factor of 1.8 for a 2.3mm thick centre-cracked panel specimen. A factor of 2.4 for a 6mm thick specimen, therefore, does not seem unreasonable. However, it is clear that additional data are required, for a range of ΔK and R , in order to establish whether these crack growth data can be explained in terms of ΔK_{eff} measured by caustics.

ii) Single Tensile Overload

Figure 6 shows the plot of crack growth rate against crack length (measured with a travelling microscope) highlighting the familiar overload induced crack retardation. It should be noted that the apparent plateau in the retardation is an anomaly of this particular set of test results and has not been observed in other similar retardation plots. Also shown in the Figure are the points where the caustic measurements were taken.

Figure 7 shows the four sets of caustic measurements over the same fatigue cycle. Remarkably, despite the dramatic retardation, the caustic results exhibit no discernible difference. Any transient change in crack closure caused by the overload did not significantly affect the crack tip stress/strain distribution where the caustics were measured, several millimetres from the crack tip.

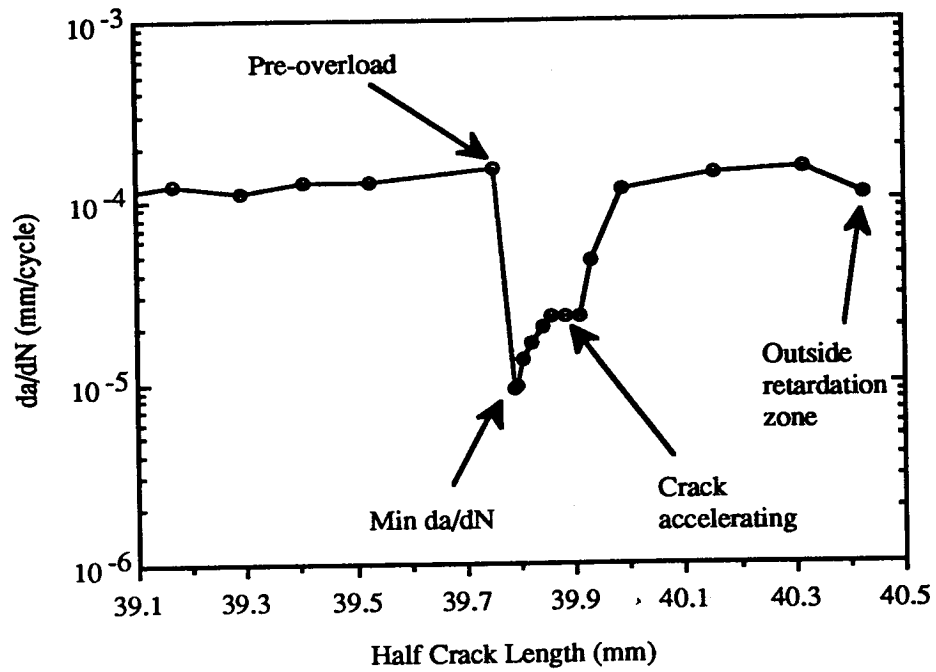


Figure 6. Overload induced crack growth retardation.

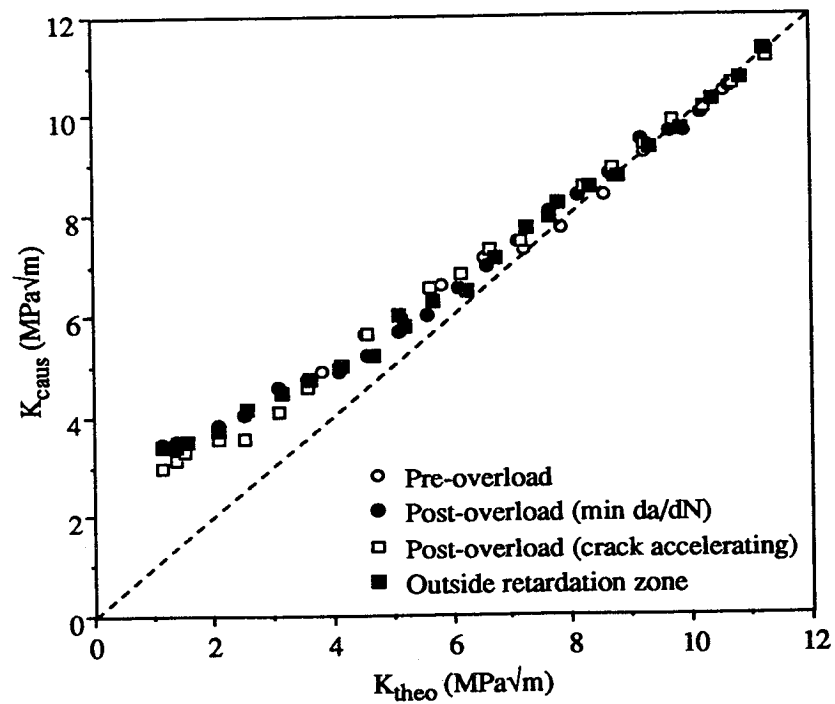


Figure 7 : Results of caustic measurements pre- and post-overload

iii) Single Compressive underload

Figure 8 shows the results following a compressive underload. A notable point is that caustics were observed even under applied compression indicating a tensile field ahead of the crack tip. The first cycle post compression shows a distinct reduction in closure, presumably caused by compressive yielding of the wake during the underload. This level of closure is maintained until over 1mm of crack growth has occurred when the results show a return to the pre-compression level. During the test, however, the crack growth rate showed no discernible change following the compression. This test indicates that the crack tip stress field was modified by a compressive underload but the crack driving force is unaffected.

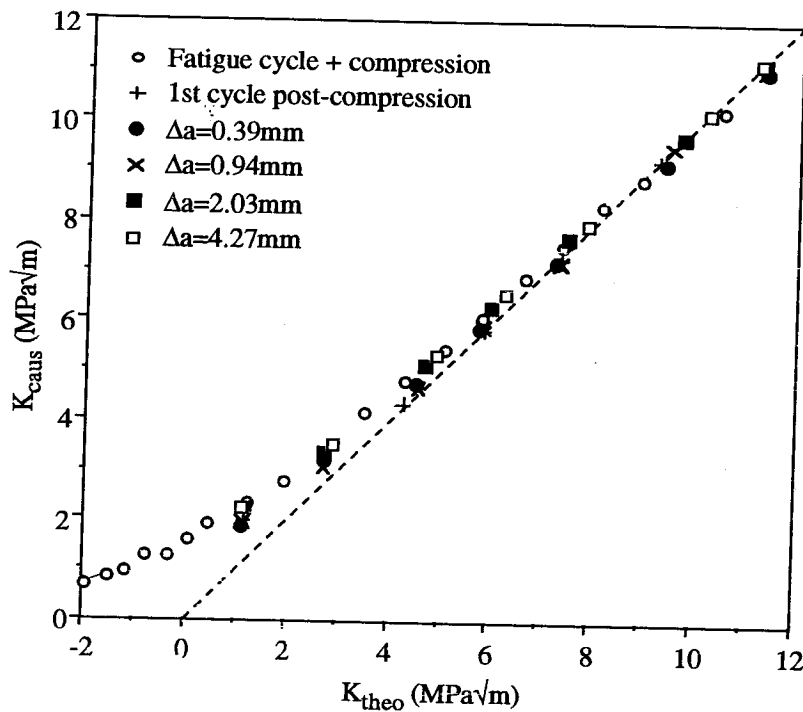


Figure 8 : Caustic results following a compressive underload

iv) High/Low Block Loading

Measurements at a low stress ratio following fatigue at a high stress ratio exaggerate the closure effects. Here, a stress ratio of 0.6 is followed by a ratio of 0.1 which is expected to cause crack closure for the whole of the lower block fatigue cycle. Indeed, the crack arrested for 100,000 cycles after which the test was ceased. Figure 9 illustrates the caustic results at selected stress intensity factors plotted (as in Figure 4) as the ratio of measured to theoretical K against the normalised radius, r_0/d .

Since, under these conditions, crack closure was anticipated to occur at $>11\text{MPa}\sqrt{\text{m}}$ the caustic curves for the whole lower block fatigue cycle were expected to follow the form of the general curve of Figure 4 (also shown in Figure 9) but with $K_{\text{caus}}/K_{\text{theo}}$ ratios indicating the same K_{caus} value $>11\text{MPa}\sqrt{\text{m}}$. The fact that the results do not follow the general curve suggests that the stress field ahead of the crack under these

conditions is not a $1/\sqrt{r}$ K-field. The curves level off at large distances from the crack tip showing the curves asymptote to a $1/\sqrt{r}$ field but closer to the tip the stress gradient is steeper than that for a $1/\sqrt{r}$ field.

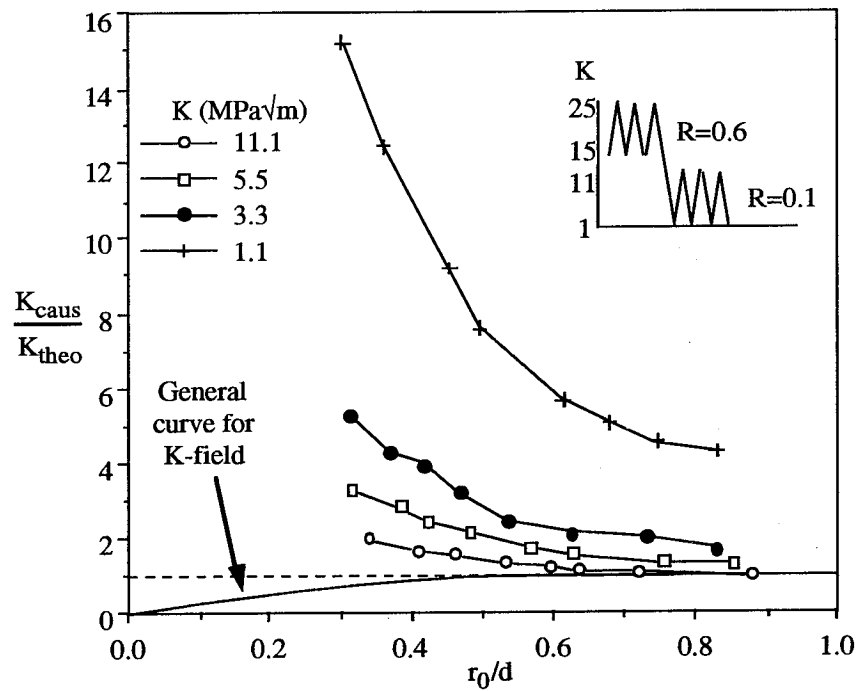


Figure 9. Caustic measurements over a range of radii.

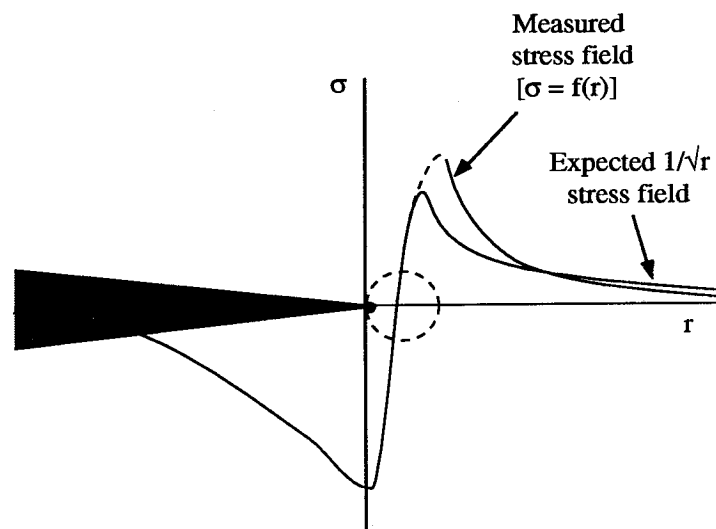


Figure 10. Schematic of the measured and expected closure stress fields

The measured stress field is shown schematically in Figure 10 which may be represented as

$$\sigma = f(r^{-n}) \quad \text{where } n > 0.5 \quad (11)$$

This strongly suggests that the simple stress intensity factor concept cannot be applied to closed cracks under the conditions investigated.

CONCLUSIONS

The caustic method has been successfully extended to enable stress intensity factors as low as $1 \text{ MPa}\sqrt{\text{m}}$ to be determined accurately for central fatigue cracks in aluminium alloy test panels. Comparisons have been made between measured values of stress intensity factor, K_{caus} , and corresponding theoretical values, K_{theo} , for a range of fatigue cracks grown under different loading conditions. In all cases, the values of K_{caus} and K_{theo} were in good agreement at maximum stress where the cracks are fully open. However, at minimum stress K_{caus} is greater than K_{theo} due to crack closure. For fatigue cracks grown under constant amplitude loading at R ratios of 0.1, 0.3 and 0.6, the difference between K_{caus} and K_{theo} was greatest for $R=0.1$ and least for $R=0.6$, in agreement with the widely reported trend that crack closure increases as R decreases. When a tensile overload was introduced during $R=0.1$ loading, no significant change in K_{caus} was observed, even though the crack growth rate was greatly reduced. In contrast, a compressive underload resulted in a change of K_{caus} , while a high/low block loading resulted in severe crack closure and complex K_{caus} data.

In general the measured values of ΔK_{eff} could not account for the observed variations of crack growth rate with loading conditions. It is concluded that the experimental values of ΔK_{eff} , determined from caustic measurements in a $1/\sqrt{r}$ stress field well outside the plastic zone, do not fully reflect local conditions which control crack growth.

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